

STUDY OF THE NUMERICAL INTEGRATION RULES USING C-LANGUAGE

A Project Report

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BONAFIDE CERTIFICATE

Certified that this project STUDY OF THE NUMERICAL INTEGRATION RULES USING C-PROGRAM is the bonafide work of V.SAI TEJA (Y183099021), M.KARUNA NAIK (Y183099017), M.SARADA (Y183099014), SK.SALMA (Y183099020) and K.CHAITANYA SAI (Y183099016) who carried out the project work under my supervision.

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PROJECT APPROVAL SHEET

Following team has done the appropriate work related to the THE STUDY OF NUMERICAL INTEGRATION RULES USING C-PROGRAM and is being submitted to SGK GOVERNMENT DEGREE COLLEGE, VINUKONDA-522647.

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CHAPTER 1

NEWTON-COTE'S QUADRATURE FORMULA

INTRODUCTION

To find the definite integral, usually we use the fundamental theorem of calculus, where we have to apply the antiderivative techniques of integration. However, sometimes, it isn't easy to find the antiderivative of an integral, like in Scientific experiments, where the function has to be determined from the observed readings. Therefore, numerical methods are used to approximate the integral in such conditions.

We know that a definite integral of the form $\int_a^b f(x)dx$ represents the area under the curve $y = f(x)$, enclosed between the limits $x = a$ and $x = b$. This integration is possible only if $f(x)$ is explicitly given and if it is integrable. The problem of numerical integration can be stated as follows:

Given a set of $(n + 1)$ data points $(x_i, y_i), i = 0, 1, 2, \dots, n$ of the function $y = f(x)$, where $f(x)$ is not known explicitly, it is required to evaluate $\int_{x_0}^{x_n} f(x)dx$.

The problem of numerical integration, like that of numerical differentiation is solved by replacing $f(x)$ with an interpolating polynomial $P_n(x)$ and obtaining $\int_{x_0}^{x_n} P_n(x)dx$ which is approximately taken as the value of $\int_{x_0}^{x_n} f(x)dx$. Numerical Integration is also known as Numerical Quadrature.

NEWTON-COTE'S QUADRATURE FORMULA

This is also known as General Quadrature formula and is the most popular and widely used numerical integration formula. It forms the basis for a number of numerical integration methods known as Newton-Cote's methods.

Derivation of Newton-Cotes formula

Let the interval $[a, b]$ be divided into n equal subintervals with interval of differencing h such that $a = x_0 < x_1 < x_2 < \dots < x_n = b$. Then $x_n = x_0 + nh$.

Newton's forward interpolation formula is

$$\begin{aligned} y(x) &= y(x_0 + uh) = P_n(x) \\ &= y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \dots \dots \dots \end{aligned} \quad (1)$$

where $u = \frac{x-x_0}{h}$.

Now, instead of $f(x)$, replace it by this interpolating polynomial.

$$\begin{aligned} \therefore \int_{x_0}^{x_n} f(x)dx &= \int_{x_0}^{x_n} P_n(x)dx \text{ where } P_n(x) \text{ is an interpolating polynomial of degree } n. \\ &= \int_{x_0}^{x_0+nh} P_n(x)dx \\ &= \int_{x_0}^{x_0+nh} \left[y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \dots \dots \dots \right] dx \end{aligned}$$

Since $x_n = x_0 + uh$, we have $dx = h.dp$ and the integration limits changes from 0 to n . Therefore, the above integral becomes

$$\begin{aligned}
 \int_{x_0}^{x_n} f(x)dx &= h \int_0^n \left[y_0 + u\Delta y_0 + \frac{u(u-1)}{2!} \Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!} \Delta^3 y_0 + \dots \right] du \\
 &= h \int_0^n \left[y_0 + u\Delta y_0 + \frac{u^2-1}{2!} \Delta^2 y_0 + \frac{u^3-3u^2+2u}{3!} \Delta^3 y_0 + \dots \right] du \\
 &= h \left[uy_0 + \frac{u^2}{2} \Delta y_0 + \left(\frac{u^3}{6} - \frac{u^2}{4} \right) \Delta^2 y_0 + \left(\frac{u^4}{24} - \frac{u^3}{6} + \frac{u^2}{6} \right) \Delta^3 y_0 + \dots \right]_0^n \\
 &= h \left[ny_0 + \frac{n^2}{2} \Delta y_0 + \left(\frac{n^3}{6} - \frac{n^2}{4} \right) \Delta^2 y_0 + \left(\frac{n^4}{24} - \frac{n^3}{6} + \frac{n^2}{6} \right) \Delta^3 y_0 \right. \\
 &\quad \left. + \left(\frac{n^5}{120} - \frac{3n^4}{48} + \frac{11n^3}{72} - \frac{n^2}{8} \right) \Delta^4 y_0 \dots \right] \tag{2}
 \end{aligned}$$

This is called Newton-Cote's Quadrature formula. From this general formula, we get different integration formulae by putting $n = 1, 2, 3, \dots$

With $n = 1$, we get *Trapezoidal rule*

With $n = 2$, we get *Simpson's $\frac{1}{3}$ rule*

With $n = 3$, we get *Simpson's $\frac{3}{8}$ rule*

With $n = 4$, we get *Boole's rule*

With $n = 6$, we get *Weddle's rule*

Note: The software used for c-program is Code Blocks.

CHAPTER 2

TRAPEZOIDAL RULE WITH C - PROGRAM

Trapezoidal Rule is one of the important integration rules. The name trapezoidal is because when the area under the curve is evaluated, then the total area is divided into small trapezoids instead of rectangles. This rule is used for approximating the definite integrals where it uses the linear approximations of the functions.

Here the function $f(x)$ is approximated by a first-order polynomial $P_1(x)$ which passes through two points.

Taking $n = 1$, in the General Quadrature formula, all differences higher than the first order will become zero and thus we get

$$\int_{x_0}^{x_1} f(x)dx = \int_{x_0}^{x_0+h} f(x)dx = h \left[y_0 + \frac{1}{2} \Delta y_0 \right] = h \left[y_0 + \frac{1}{2} (y_1 - y_0) \right] = \frac{h}{2} (y_0 + y_1)$$

Similarly,

$$\int_{x_1}^{x_2} f(x)dx = \int_{x_0+h}^{x_0+2h} f(x)dx = h \left[y_1 + \frac{1}{2} \Delta y_1 \right] = h \left[y_1 + \frac{1}{2} (y_2 - y_1) \right] = \frac{h}{2} (y_1 + y_2)$$

$$\int_{x_2}^{x_3} f(x)dx = \int_{x_0+2h}^{x_0+3h} f(x)dx = \frac{h}{2} (y_2 + y_3)$$

.....

Finally, $\int_{x_0+(n-1)h}^{x_0+nh} f(x)dx = \frac{h}{2}(y_{n-1} + y_n)$

$$\begin{aligned}\therefore \int_{x_0}^{x_n} f(x)dx &= \int_{x_0}^{x_1} f(x)dx + \int_{x_1}^{x_2} f(x)dx + \int_{x_2}^{x_3} f(x)dx + \cdots + \int_{x_0+(n-1)h}^{x_0+nh} f(x)dx \\ &= \frac{h}{2}(y_0 + y_1) + \frac{h}{2}(y_1 + y_2) + \cdots + \frac{h}{2}(y_{n-1} + y_n) \\ &= \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + y_4 + \cdots + y_{n-1})]\end{aligned}$$

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + y_4 + \cdots + y_{n-1})]$$

OR

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{2}[(\text{Sum of the First and Last Ordinates})$$

$$+ 2(\text{Sum of the Remaining Ordinates})]$$

This is known as Trapezoidal Rule.

Geometrical Interpretation

Consider the points $P_0(x_0, y_0), P_1(x_1, y_1), P_2(x_2, y_2), \dots, P_n(x_n, y_n)$. Suppose the curve $y = f(x)$ passing through the above points be approximated by the union of the line segments joining $(P_0, P_1), (P_1, P_2), (P_2, P_3), \dots, (P_{n-1}, P_n)$.

Geometrically, the curve $y = f(x)$ is replaced by n straight line segments joining the points $P_0(x_0, y_0)$ and $P_1(x_1, y_1)$; $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$; $\dots$ $P_{n-1}(x_{n-1}, y_{n-1})$ and $P_n(x_n, y_n)$. The area bounded by the curve $y = f(x)$, x -axis and the ordinates $x = x_0$ and $x = x_n$ is then approximately equal to the sum of the areas of the n trapeziums.

Though this method is very simple for calculation purposes of numerical integration, the error in this case is significant. The accuracy of the result can be improved by increasing the number of intervals or by decreasing the value of h .

Example Evaluate $\int_0^1 \frac{1}{1+x} dx$ by using Trapezoidal rule with six sub-intervals.

Solution Divide the interval $[0,1]$ into six subintervals.

$$\text{Here } h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$$

The values of x and y are tabulated as below:

x	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1
$y = \frac{1}{1+x}$	1	0.857142	0.75	0.66667	0.6	0.545454	0.5

$$\therefore y_0 = 1, y_1 = 0.8571, y_2 = 0.75, y_3 = 0.6667, y_4 = 0.6, y_5 = 0.5454, y_6 = 0.5$$

By Trapezoidal Rule,

$$\begin{aligned} \int_0^1 \frac{1}{1+x} dx &= \frac{h}{2} [(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)] \\ &= \frac{1}{12} [(1 + 0.5) + 2(0.857142 + 0.75 + 0.666667 + 0.6 + 0.545454)] \\ &= 0.694877 \end{aligned}$$

Analytical Solution: $\int_0^1 \frac{1}{1+x} dx = [\log(1+x)]_0^1 = \ln 2 = 0.693147$

TRAPEZOIDAL RULE C-PROGRAM

```
#include<stdio.h>
#include<math.h>

double f(double x)
{
    return 1/(1+x);
}

main()
{
    int n,i;
    double a,b,h,x,sum=0,integral;

    printf("\nEnter the no. of sub-intervals 'n': ");
    scanf("%d",&n);

    printf("\nEnter the lower limit 'a': ");
    scanf("%lf",&a);

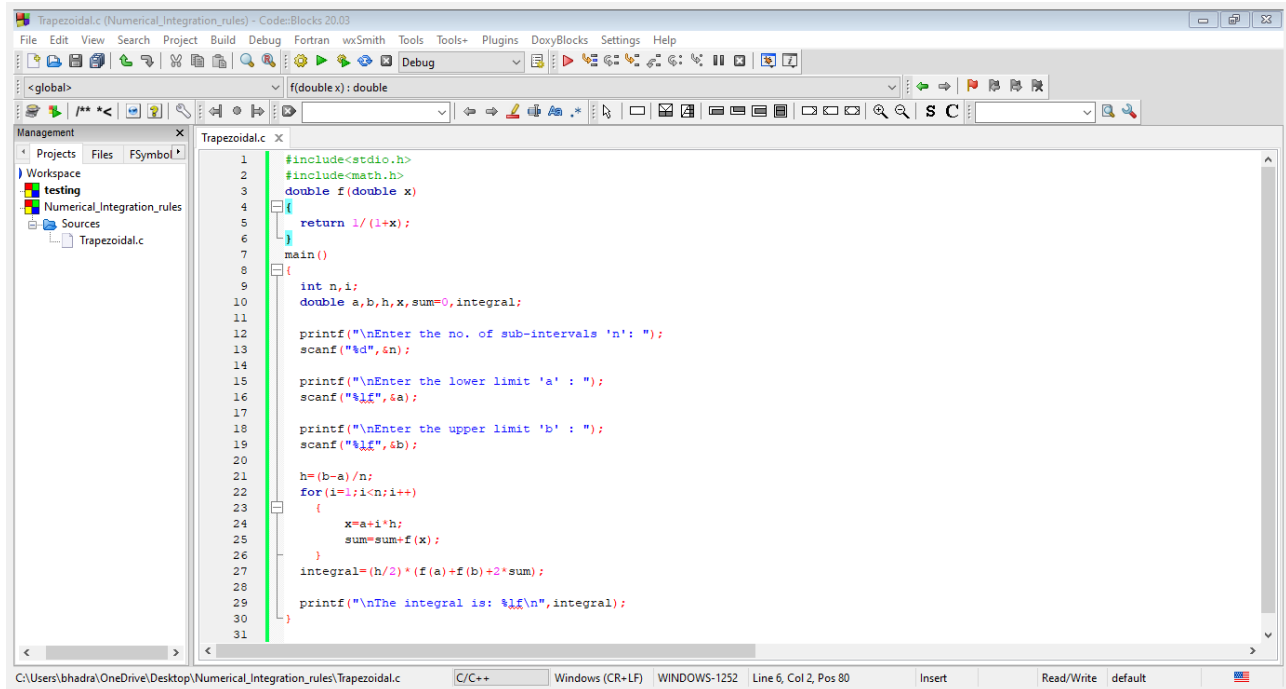
    printf("\nEnter the upper limit 'b': ");
    scanf("%lf",&b);

    h=(b-a)/n;
    for(i=1;i<n;i++)
    {
        x=a+i*h;
        sum=sum+f(x);
    }
    integral=(h/2)*(f(a)+f(b)+2*sum);

    printf("\nThe integral is: %lf\n",integral);
}

===== END OF THE PROGRAM =====
```

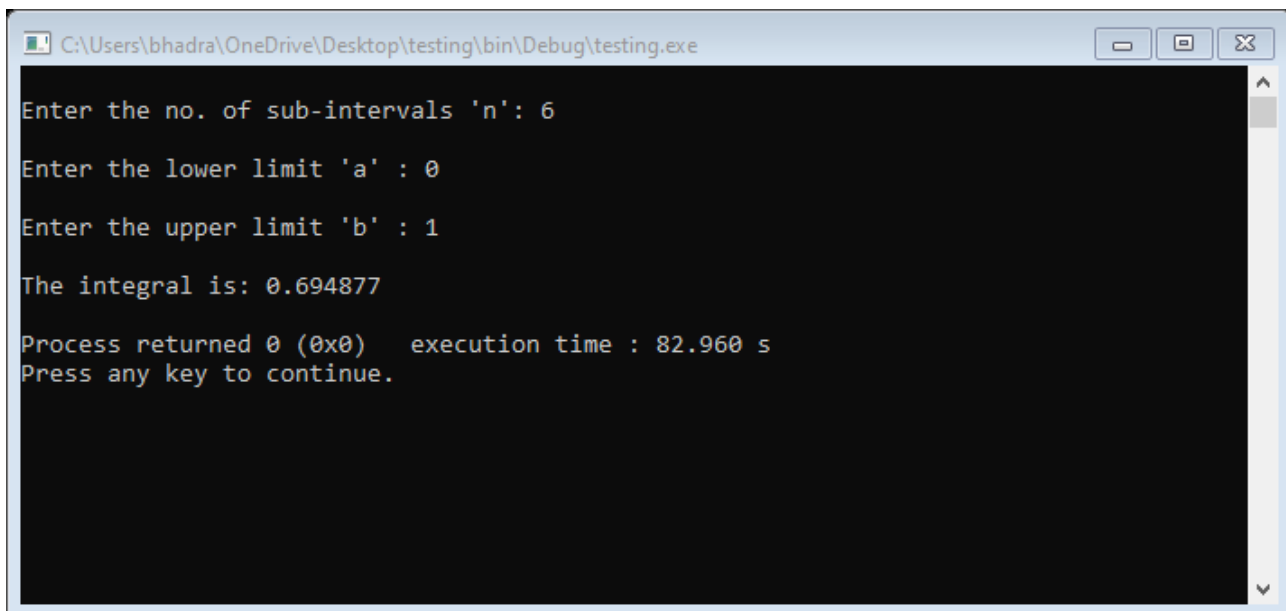
The screenshot of the c program for trapezoidal rule:



The screenshot shows a C code editor with the following code:

```
1  #include<stdio.h>
2  #include<math.h>
3  double f(double x)
4  {
5      return 1/(1+x);
6  }
7  main()
8  {
9      int n,i;
10     double a,b,h,x,sum=0,integral;
11
12     printf("\nEnter the no. of sub-intervals 'n': ");
13     scanf("%d",&n);
14
15     printf("\nEnter the lower limit 'a' : ");
16     scanf("%lf",&a);
17
18     printf("\nEnter the upper limit 'b' : ");
19     scanf("%lf",&b);
20
21     h=(b-a)/n;
22     for(i=1;i<n;i++)
23     {
24         x=a+i*h;
25         sum=sum+f(x);
26     }
27     integral=(h/2)*(f(a)+f(b)+2*sum);
28
29     printf("\nThe integral is: %lf\n",integral);
30 }
31
```

The screenshot of the output with $n = 6, a = 0, b = 1$ which gives the integral 0.694877, which matches exactly with the answer in the above example:



The screenshot shows a terminal window with the following output:

```
Enter the no. of sub-intervals 'n': 6
Enter the lower limit 'a' : 0
Enter the upper limit 'b' : 1
The integral is: 0.694877
Process returned 0 (0x0)   execution time : 82.960 s
Press any key to continue.
```

From the below output screenshots, we can conclude that the exact value of the integral is 0.693147 which is obtained by increasing the number of sub-intervals.

```
C:\Users\bhadra\OneDrive\Desktop\testing\bin\Debug\tes...
Enter the no. of sub-intervals 'n': 100
Enter the lower limit 'a' : 0
Enter the upper limit 'b' : 1
The integral is: 0.693153
Process returned 0 (0x0)   execution time : 4.334 s
Press any key to continue.
```

```
C:\Users\bhadra\OneDrive\Desktop\testing\bin\Debug\testing...
Enter the no. of sub-intervals 'n': 500
Enter the lower limit 'a' : 0
Enter the upper limit 'b' : 1
The integral is: 0.693147
Process returned 0 (0x0)   execution time : 12.981 s
Press any key to continue.
```

```
C:\Users\bhadra\OneDrive\Desktop\testing\bin\Debug\testing.e...
Enter the no. of sub-intervals 'n': 1000
Enter the lower limit 'a' : 0
Enter the upper limit 'b' : 1
The integral is: 0.693147
Process returned 0 (0x0)   execution time : 7.575 s
Press any key to continue.
```

CHAPTER 3

SIMPSON'S RULES WITH C - PROGRAMS

We have Simpson's $\frac{1}{3}$ -rule and Simpson's $\frac{3}{8}$ -rule.

Simpson's $\frac{1}{3}$ -rule

In Simpson's $\frac{1}{3}$ -rule, the function $f(x)$ is approximated by a second order polynomial $P_2(x)$ which passes through three successive points.

Taking $n = 2$ in the the General Quadrature formula, by replacing the curve $y = f(x)$ by $\frac{n}{2}$ parabolas, all differences higher than the second order will become zero and thus we get

$$\begin{aligned}\int_{x_0}^{x_2} f(x)dx &= \int_{x_0}^{x_0+2h} f(x)dx = 2h \left[y_0 + \Delta y_0 + \frac{1}{6} \Delta^2 y_0 \right] \\ &= 2h \left[y_0 + (y_1 - y_0) + \frac{1}{6} (y_2 - 2y_1 + y_0) \right] \\ &= \frac{h}{3} (y_0 + 4y_1 + y_2)\end{aligned}$$

Similarly,

$$\int_{x_2}^{x_4} f(x)dx = \frac{h}{3} (y_2 + 4y_3 + y_4)$$

$$\int_{x_4}^{x_6} f(x)dx = \frac{h}{3}(y_4 + 4y_5 + y_6)$$

.....

$$\int_{x_{n-2}}^{x_n} f(x)dx = \frac{h}{3}(y_{n-2} + 4y_{n-1} + y_n)$$

Adding all these integrals, we get

$$\begin{aligned} \int_{x_0}^{x_n} f(x)dx &= \int_{x_0}^{x_2} f(x)dx + \int_{x_2}^{x_4} f(x)dx + \int_{x_4}^{x_6} f(x)dx + \cdots + \int_{x_{n-2}}^{x_n} f(x)dx \\ &= \frac{h}{3}(y_0 + 4y_1 + y_2) + \frac{h}{3}(y_2 + 4y_3 + y_4) + \cdots + \frac{h}{3}(y_{n-2} + 4y_{n-1} + y_n) \\ &= \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + \cdots + y_{n-1}) + 2(y_2 + y_4 + \cdots + y_{n-2})] \end{aligned}$$

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + \cdots + y_{n-1}) + 2(y_2 + y_4 + \cdots + y_{n-2})]$$

OR

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{3}[(\text{Sum of the First and Last Ordinates})$$

$$+ 4(\text{Sum of the Odd Ordinates}) + 2(\text{Sum of the remaining Even Ordinates})]$$

with the convention that $y_1, y_3, \dots, y_{n-1}$ are odd ordinates and $y_0, y_2, y_4, \dots, y_{n-2}, y_n$ are even ordinates.

This is known as *Simpsons $\frac{1}{3}$ – rule*. The number of intervals in this rule must be even.

Example Evaluate $\int_0^1 \frac{1}{1+x} dx$ by using Simpson's $\frac{1}{3}$ —rule with six sub-intervals.

Solution Divide the interval $[0,1]$ into six subintervals.

$$\text{Here } h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$$

The values of x and y are tabulated as below:

x	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1
$y = \frac{1}{1+x}$	1	0.857142	0.75	0.66667	0.6	0.545454	0.5

$$\therefore y_0 = 1, y_1 = 0.857142, y_2 = 0.75, y_3 = 0.66667, y_4 = 0.6, y_5 = 0.545454, y_6 = 0.5$$

By Simpson's $\frac{1}{3}$ —rule,

$$\int_0^1 \frac{1}{1+x} dx = \frac{h}{3} [(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$$

$$= \frac{1}{18} [(1 + 0.5) + 4(0.857142 + 0.666667 + 0.545454) + 2(0.75 + 0.6)]$$

$$= 0.69316956$$

SIMPSON'S ONE-THIRD RULE C-PROGRAM

```
#include<stdio.h>
#include<math.h>

double f(double x)
{
    return 1/(1+x);
}

main()
{
    int n,i;
    double a,b,h,x,sum=0,integral;

    printf("\nEnter the no. of sub-intervals 'n' (EVEN): ");
    scanf("%d",&n);

    if (n%2==0)
    {
        printf("\nEnter the lower limit 'a': ");
        scanf("%lf",&a);

        printf("\nEnter the upper limit 'b': ");
        scanf("%lf",&b);

        h=(b-a)/n;

        for(i=1;i<n;i++)
        {
            x=a+i*h;
            if(i%2==0)
            {
                sum=sum+2*f(x);
            }
        }
    }
}
```

```

        else
        {
            sum=sum+4*f(x);
        }
    }
    integral=(h/3)*(f(a)+f(b)+sum);

    printf("\nThe integral is: %lf\n",integral);

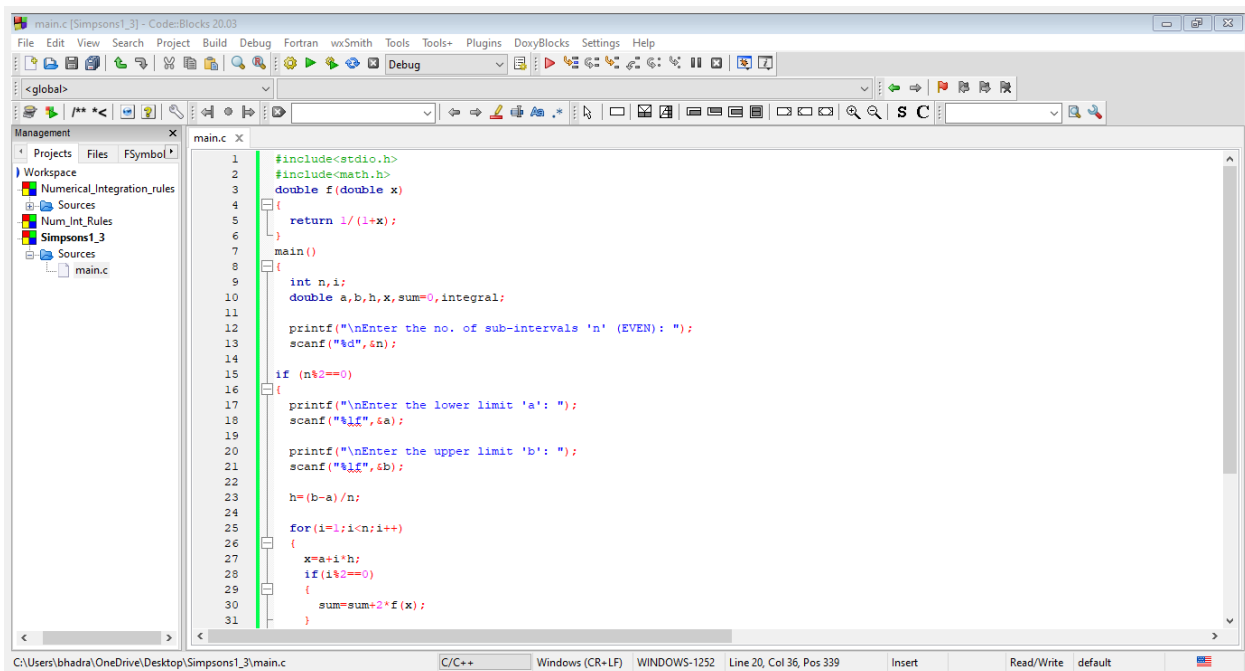
}

else
{
    printf("Oops!\n Simpons one-third rule is not applicable.\n
Please enter even number.\n");
}
}

```

===== END OF THE PROGRAM =====

The screenshots of the c program for Simpson's $\frac{1}{3}$ - rule in the CodeBlocks software:



```

1  #include<stdio.h>
2  #include<math.h>
3  double f(double x)
4  {
5      return 1/(1+x);
6  }
7  main()
8  {
9      int n,i;
10     double a,b,h,x,sum=0,integral;
11
12     printf("\nEnter the no. of sub-intervals 'n' (EVEN): ");
13     scanf("%d",&n);
14
15     if (n%2==0)
16     {
17         printf("\nEnter the lower limit 'a': ");
18         scanf("%lf",&a);
19
20         printf("\nEnter the upper limit 'b': ");
21         scanf("%lf",&b);
22
23         h=(b-a)/n;
24
25         for(i=1;i<n;i++)
26         {
27             x=a+i*h;
28             if(i%2==0)
29             {
30                 sum=sum+2*f(x);
31             }

```

The screenshot shows the Code::Blocks IDE with the file 'main.c' open. The code implements Simpson's 1/3 rule for numerical integration. It prompts the user for the number of sub-intervals 'n' (must be even), the lower limit 'a', and the upper limit 'b'. It then calculates the integral using the formula $\int_a^b f(x) dx \approx \frac{h}{3} [f(a) + 4f(x_1) + 2f(x_2) + \dots + 4f(x_{n-1}) + f(b)]$, where $h = (b-a)/n$. The output shows the integral value for $n=6$.

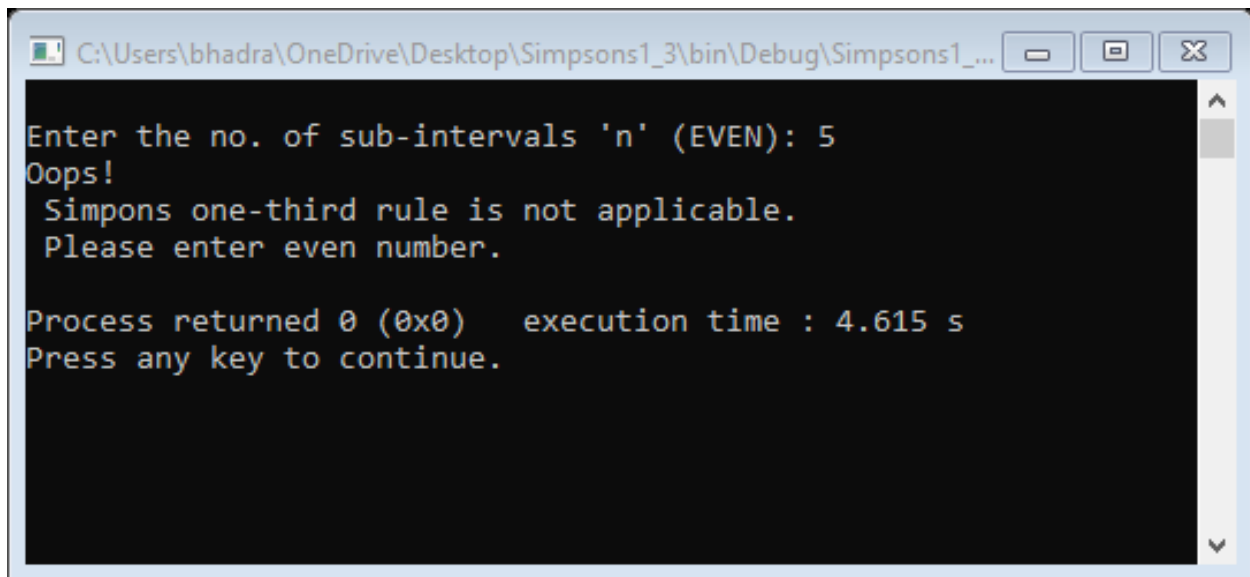
```
14  if (n%2==0)
15  {
16      printf("\nEnter the lower limit 'a': ");
17      scanf("%lf",&a);
18
19      printf("\nEnter the upper limit 'b': ");
20      scanf("%lf",&b);
21
22      h=(b-a)/n;
23
24      for (i=1;i<n;i++)
25      {
26          x=a+i*h;
27          if(i%2==0)
28          {
29              sum=sum+2*f(x);
30          }
31          else
32          {
33              sum=sum+4*f(x);
34          }
35      }
36      integral=(h/3)*(f(a)+f(b)+sum);
37      printf("\nThe integral is: %lf\n",integral);
38  }
39  else
40  {
41      printf("Oops!\n Simpson's one-third rule is not applicable.\n Please enter even number.\n");
42  }
43  }
```

The integral value with $n = 6$, showing the output:

The screenshot shows a Windows command prompt window titled 'C:\Users\bhadra\OneDrive\Desktop\Simpsons1_3\bin\Debug\Simpsons1_3.exe'. The program's output is as follows:

```
Enter the no. of sub-intervals 'n' (EVEN): 6
Enter the lower limit 'a': 0
Enter the upper limit 'b': 1
The integral is: 0.693170
Process returned 0 (0x0)   execution time : 6.622 s
Press any key to continue.
```

Simpson's $\frac{1}{3}$ -rule is not applicable if n is not an even number:

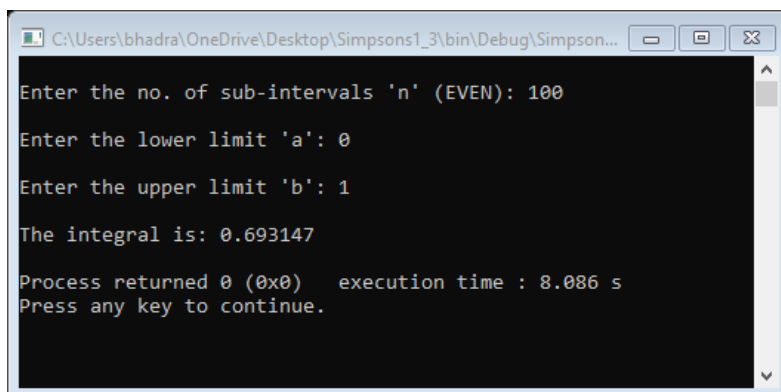


```
C:\Users\bhadra\OneDrive\Desktop\Simpsons1_3\bin\Debug\Simpsons1_3.exe

Enter the no. of sub-intervals 'n' (EVEN): 5
Oops!
Simpsons one-third rule is not applicable.
Please enter even number.

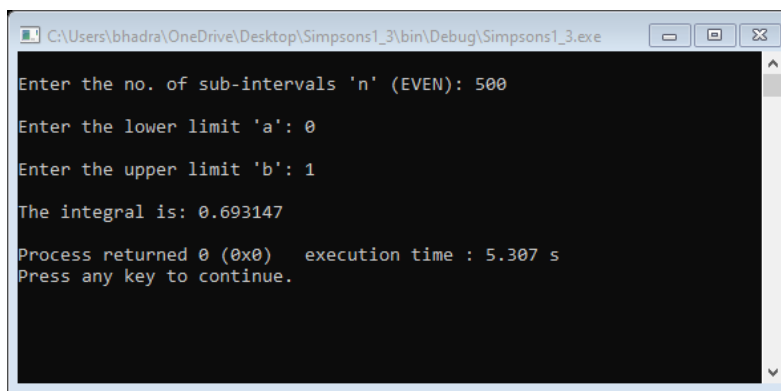
Process returned 0 (0x0)   execution time : 4.615 s
Press any key to continue.
```

From the below output screenshots, we can conclude that the exact value of the integral is 0.693147 which is obtained by increasing the number of sub-intervals.



```
C:\Users\bhadra\OneDrive\Desktop\Simpsons1_3\bin\Debug\Simpson...

Enter the no. of sub-intervals 'n' (EVEN): 100
Enter the lower limit 'a': 0
Enter the upper limit 'b': 1
The integral is: 0.693147
Process returned 0 (0x0)   execution time : 8.086 s
Press any key to continue.
```



```
C:\Users\bhadra\OneDrive\Desktop\Simpsons1_3\bin\Debug\Simpsons1_3.exe

Enter the no. of sub-intervals 'n' (EVEN): 500
Enter the lower limit 'a': 0
Enter the upper limit 'b': 1
The integral is: 0.693147
Process returned 0 (0x0)   execution time : 5.307 s
Press any key to continue.
```

Simpson's $\frac{3}{8}$ —rule

In Simpson's $\frac{3}{8}$ —rule, the function $f(x)$ is approximated by a third order polynomial $P_3(x)$ which passes through four successive points.

Taking $n = 3$ in the the General Quadrature formula, all differences higher than the third order will become zero and thus we get

$$\begin{aligned}\int_{x_0}^{x_3} f(x)dx &= 3h \left[y_0 + \frac{3}{2}\Delta y_0 + \frac{3}{4}\Delta^2 y_0 + \frac{1}{8}\Delta^3 y_0 \right] \\ &= 3h \left[y_0 + \frac{3}{2}(y_1 - y_0) + \frac{3}{4}(y_2 - 2y_1 + y_0) + \frac{1}{8}(y_3 - 3y_2 + 3y_1 - y_0) \right] \\ &= \frac{3h}{8}(y_0 + 3y_1 + 3y_2 + y_3)\end{aligned}$$

Similarly, $\int_{x_3}^{x_6} f(x)dx = \frac{3h}{8}(y_3 + 3y_4 + 3y_5 + y_6)$

.....

$$\int_{x_{n-3}}^{x_n} f(x)dx = \frac{3h}{8}(y_{n-3} + 3y_{n-2} + 3y_{n-1} + y_n)$$

Adding all these integrals, we get

$$\begin{aligned}\int_{x_0}^{x_n} f(x)dx &= \int_{x_0}^{x_3} f(x)dx + \int_{x_3}^{x_6} f(x)dx + \cdots + \int_{x_{n-3}}^{x_n} f(x)dx \\ &= \frac{3h}{8} [(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + \cdots + y_{n-1}) \\ &\quad + 2(y_3 + y_6 + \cdots + y_{n-3})]\end{aligned}$$

$$\int_{x_0}^{x_n} f(x) dx = \frac{3h}{8} [(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + \cdots + y_{n-1}) + 2(y_3 + y_6 + \cdots + y_{n-3})]$$

This is known as *Simpson's $\frac{3}{8}$ – rule*. The number of intervals in this rule must a multiple of 3.

Example Evaluate $\int_0^1 \frac{1}{1+x} dx$ by using Simpson's $\frac{3}{8}$ –rule with six sub-intervals.

Solution Divide the interval $[0,1]$ into six subintervals.

$$\text{Here } h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$$

The values of x and y are tabulated as below:

x	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1
$y = \frac{1}{1+x}$	1	0.857142	0.75	0.666667	0.6	0.545454	0.5

$$\therefore y_0 = 1, y_1 = 0.857142, y_2 = 0.75, y_3 = 0.666667, y_4 = 0.6, y_5 = 0.545454, y_6 = 0.5$$

$$\text{By Simpson's } \frac{3}{8} \text{ –rule, } \int_0^1 \frac{1}{1+x} dx = \frac{3h}{8} [(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3)]$$

$$= \frac{3}{48} [(1 + 0.5) + 3(0.857142 + 0.75 + 0.6 + 0.545454) + 2(0.666667)]$$

$$= 0.69319513$$

SIMPSON'S THREE-EIGHT RULE C-PROGRAM

```
#include<stdio.h>
#include<math.h>

double f(double x)
{
    return 1/(1+x);
}

main()
{
    int n,i;
    double a,b,h,x,sum=0,integral;

    printf("\nEnter the no. of sub-intervals 'n' (EVEN): ");
    scanf("%d",&n);

    if (n%3==0)
    {
        printf("\nEnter the lower limit 'a': ");
        scanf("%lf",&a);

        printf("\nEnter the upper limit 'b': ");
        scanf("%lf",&b);

        h=(b-a)/n;

        for(i=1;i<n;i++)
        {
            x=a+i*h;
            if(i%3==0)
            {
                sum=sum+2*f(x);
            }
        }
    }
}
```

```

        else
        {
            sum=sum+3*f(x);
        }
    }
    integral=(3*h/8)*(f(a)+f(b)+sum);

    printf("\nThe integral is: %lf\n",integral);

}

else
{
    printf("Oops!\n    Simpons    three-eighth    rule    is    not
    applicable.\n Please enter a number which is a multiple of 3.\n");
}
}

```

===== END OF THE PROGRAM =====

The screenshot of the c program for Simpson's one-third rule:

The screenshot shows the Code::Blocks IDE with a C program for Simpson's one-third rule. The program is named 'main.c' and is located in the 'Num_Int_Rules' project. The code is as follows:

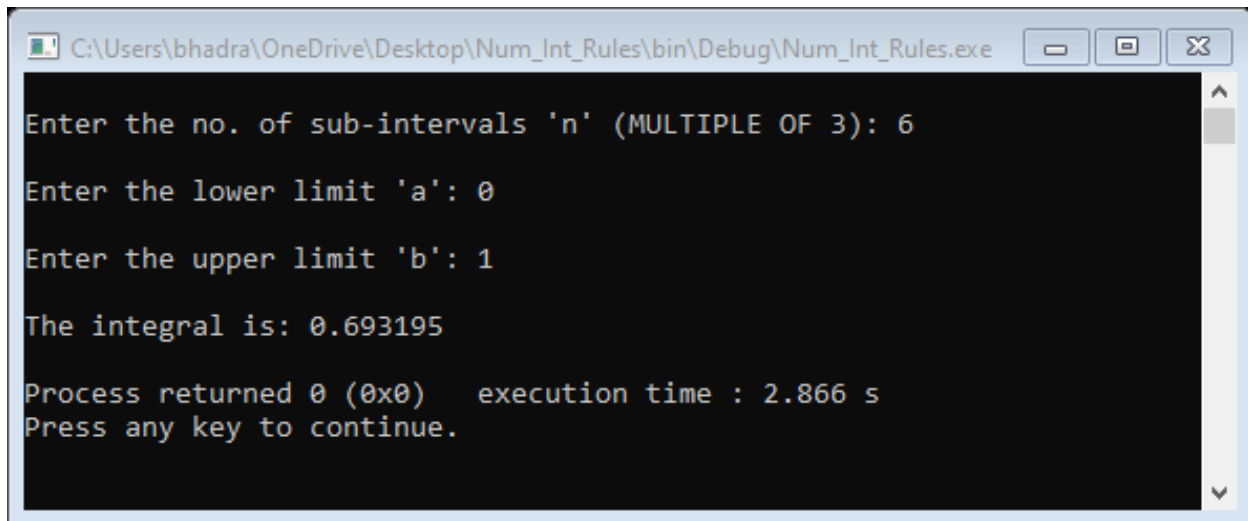
```

1  #include<stdio.h>
2  #include<math.h>
3  double f(double x)
4  {
5      return 1/(1+x);
6  }
7  main()
8  {
9      int n,i;
10     double a,b,h,x,sum=0,integral;
11
12     printf("\nEnter the no. of sub-intervals 'n' (MULTIPLE OF 3): ");
13     scanf("%d",&n);
14
15     if (n%3==0)
16     {
17         printf("\nEnter the lower limit 'a': ");
18         scanf("%lf",&a);
19
20         printf("\nEnter the upper limit 'b': ");
21         scanf("%lf",&b);
22
23         h=(b-a)/n;
24
25         for(i=1;i<n;i++)
26         {
27             x=a+i*h;
28             if(i%3==0)
29             {
30                 sum=sum+2*f(x);
31             }

```

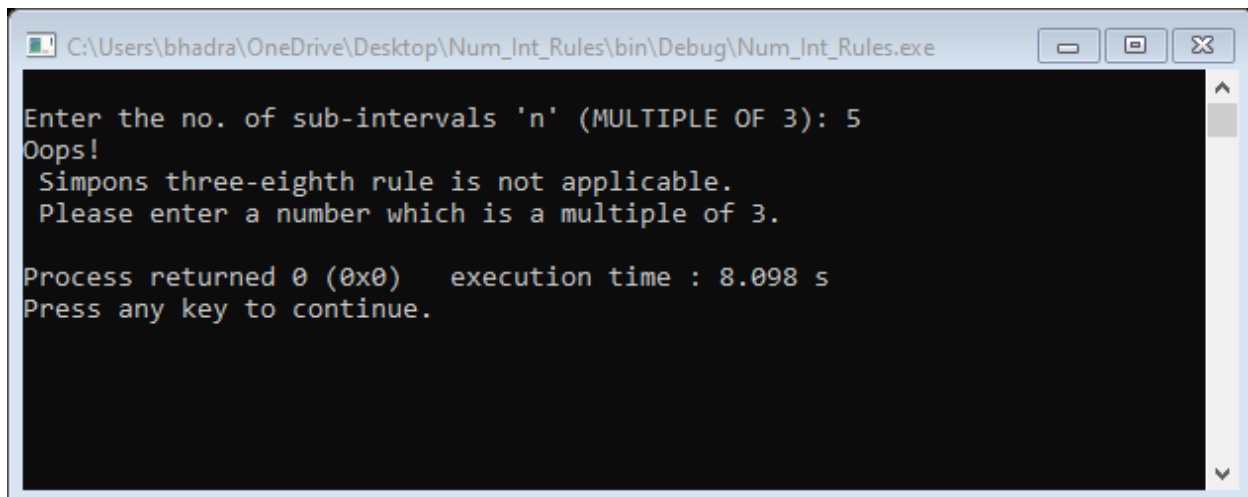
The screenshot also shows the project structure on the left, with 'Num_Int_Rules' as the active project and 'main.c' as the source file. The status bar at the bottom indicates the file is 'C/C++', the window is 'Windows (CR+LF)', and the current line is 14, column 1, position 229.

The integral value with $n=6$:



```
C:\Users\bhadra\OneDrive\Desktop\Num_Int_Rules\bin\Debug\Num_Int_Rules.exe
Enter the no. of sub-intervals 'n' (MULTIPLE OF 3): 6
Enter the lower limit 'a': 0
Enter the upper limit 'b': 1
The integral is: 0.693195
Process returned 0 (0x0)   execution time : 2.866 s
Press any key to continue.
```

Showing Simpson's three-fourth rule is not applicable if n is not a multiple of 3:



```
C:\Users\bhadra\OneDrive\Desktop\Num_Int_Rules\bin\Debug\Num_Int_Rules.exe
Enter the no. of sub-intervals 'n' (MULTIPLE OF 3): 5
Oops!
Simpons three-eighth rule is not applicable.
Please enter a number which is a multiple of 3.
Process returned 0 (0x0)   execution time : 8.098 s
Press any key to continue.
```

From the below output screenshots, we can conclude that the exact value of the integral is 0.693022 which is obtained by increasing the number of sub-intervals, but is not matching with the analytical solution. Thus Simpson's one-third rule is more accurate than the three-fourth solution.

```
C:\Users\bhadra\OneDrive\Desktop\Num_Int_Rules\bin\Debug\Num_Int_Rules.exe

Enter the no. of sub-intervals 'n' (MULTIPLE OF 3): 100
Enter the lower limit 'a': 0
Enter the upper limit 'b': 1
The integral is: 0.691894
Process returned 0 (0x0)   execution time : 6.119 s
Press any key to continue.
```

```
C:\Users\bhadra\OneDrive\Desktop\Num_Int_Rules\bin\Debug\Num_Int_Rules.exe

Enter the no. of sub-intervals 'n' (MULTIPLE OF 3): 500
Enter the lower limit 'a': 0
Enter the upper limit 'b': 1
The integral is: 0.693022
Process returned 0 (0x0)   execution time : 5.235 s
Press any key to continue.
```

```
C:\Users\bhadra\OneDrive\Desktop\Num_Int_Rules\bin\Debug\Num_Int_Rules.exe

Enter the no. of sub-intervals 'n' (MULTIPLE OF 3): 1000
Enter the lower limit 'a': 0
Enter the upper limit 'b': 1
The integral is: 0.693022
Process returned 0 (0x0)   execution time : 4.168 s
Press any key to continue.
```

CHAPTER 4

BOOLE'S RULE AND WEDDLE'S RULE WITH C - PROGRAMS

BOOLE'S RULE

Taking $n = 4$ in the the General Quadrature formula, all differences higher than the fourth order will become zero and thus we get

$$\begin{aligned} \int_{x_0}^{x_n} f(x) dx = & \frac{2h}{45} [(7y_0 + 32y_1 + 12y_2 + 32y_3 + 7y_4) \\ & + (7y_4 + 32y_5 + 12y_6 + 32y_7 + 7y_8) + \dots \\ & + (7y_{n-4} + 32y_{n-3} + 12y_{n-2} + 32y_{n-1} + 7y_n)] \end{aligned}$$

This is known as *Boole's rule*. The number of intervals in this rule must be a multiple of 4.

Example Evaluate $\int_0^1 \frac{1}{1+x} dx$ by using *Boole's rule* with four sub-intervals.

Solution Divide the interval $[0,1]$ into four subintervals.

$$\text{Here } h = \frac{b-a}{4} = \frac{1-0}{4} = \frac{1}{4}$$

The values of x and y are tabulated as below:

x	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$y = \frac{1}{1+x}$	1	0.8	0.666667	0.5714286	0.5

$$\therefore y_0 = 1, y_1 = 0.8, y_2 = 0.666667, y_3 = 0.5714286, y_4 = 0.5$$

By Boole's rule,

$$\begin{aligned}\int_0^1 \frac{1}{1+x} dx &= \frac{2h}{45} [7y_0 + 32y_1 + 12y_2 + 32y_3 + 7y_4] \\ &= \frac{1}{90} [7(1) + 32(0.8) + 12(0.666667) + 32(0.5714286) + 7(0.5)] \\ &= 0.69317466\end{aligned}$$

BOOLE'S RULE C-PROGRAM

```
#include<stdio.h>
#include<math.h>

double f(double x)
{
    return 1/(1+x);
}

main()
{
    int n,m,i;
    double a,b,h,x,sum=0,integral;

    printf("\nEnter the no. of sub-intervals 'n' (EVEN): ");
    scanf("%d",&n);

    if (n%4==0)
    {
        printf("\nEnter the lower limit 'a': ");
        scanf("%lf",&a);

        printf("\nEnter the upper limit 'b': ");
        scanf("%lf",&b);

        h=(b-a)/n;
        m=n/4;

        for(i=1;i<=m;i++)
        {
            sum=sum+7*f(a)+32*f(a+h)+12*f(a+2*h)+32*f(a+3*h)+7*f(a+4*h);
            a=a+4*h;
        }
        integral=(2*h/45)*sum;
```

```

printf("\nThe integral is: %lf\n",integral);

}

else
{
    printf("Oops!\n BOOLE'S RULE is not applicable.\n Please
enter a number which is a multiple of 4.\n");
}

}

===== END OF THE PROGRAM =====

```

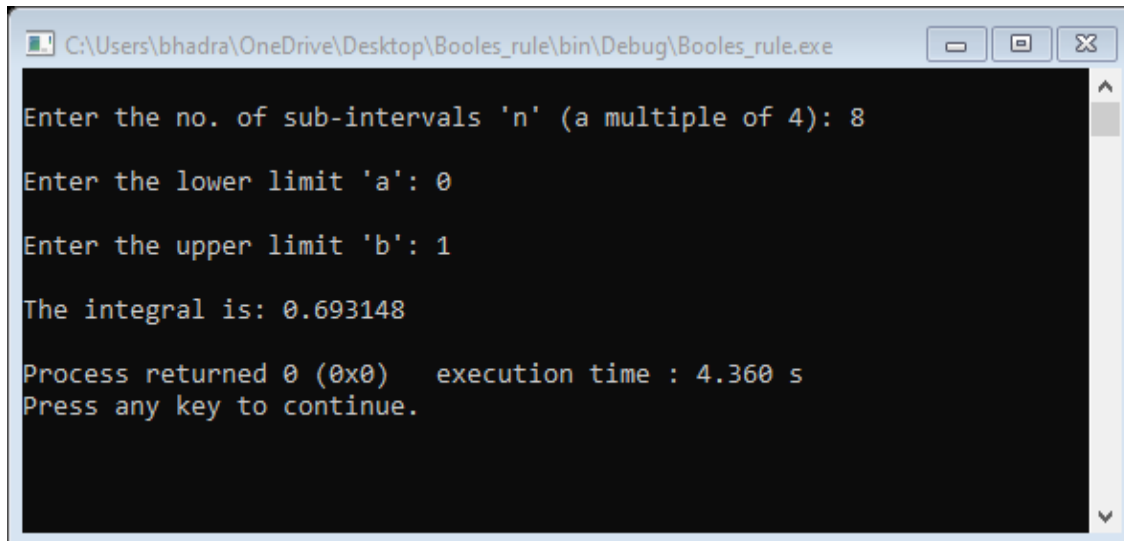
Screenshot of the c-program in CodeBlocks:

```

1  #include<stdio.h>
2  #include<math.h>
3  double f(double x)
4  {
5      return 1/(1+x);
6  }
7  main()
8  {
9      int n,m,i;
10     double a,b,h,x,sum=0,integral;
11     printf("\nEnter the no. of sub-intervals 'n' (a multiple of 4): ");
12     scanf("%d",&n);
13     if (n%4!=0)
14     {
15         printf("\nEnter the lower limit 'a': ");
16         scanf("%lf",&a);
17         printf("\nEnter the upper limit 'b': ");
18         scanf("%lf",&b);
19         h=(b-a)/n;
20         m=n/4;
21         for(i=1;i<=m;i++)
22         {
23             sum=sum+7*f(a)+32*f(a+h)+12*f(a+2*h)+32*f(a+3*h)+7*f(a+4*h);
24             a=a+4*h;
25         }
26         integral=(2*h/45)*sum;
27         printf("\nThe integral is: %lf\n",integral);
28     }
29     else
30     {
31         printf("Oops!\n BOOLE'S RULE is not applicable.\n Please enter a number which is a multiple of 4.\n");
32     }
33 }

```

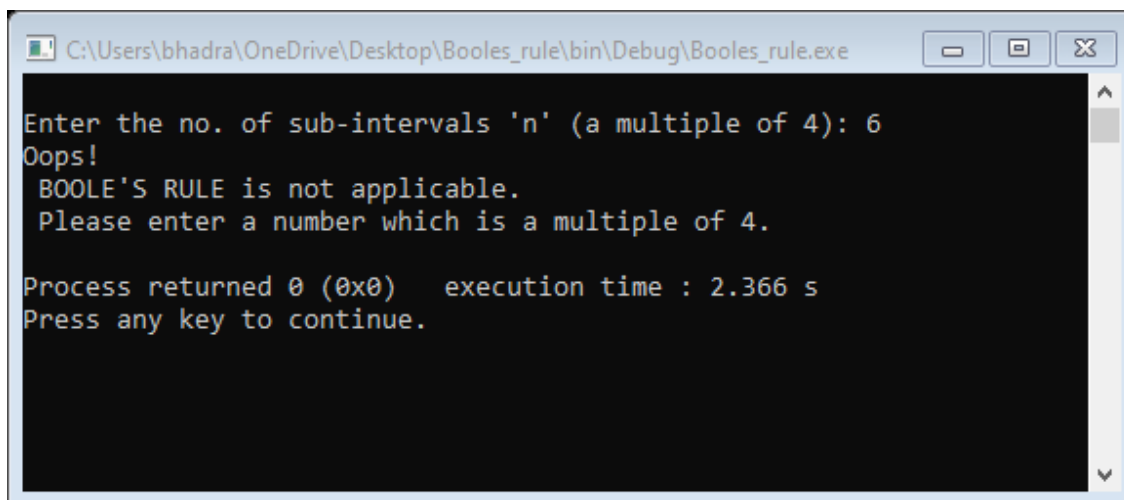
Screenshot of the output showing integral value 0.693148
with $n = 8$:



```
C:\Users\bhadra\OneDrive\Desktop\Booles_rule\bin\Debug\Booles_rule.exe

Enter the no. of sub-intervals 'n' (a multiple of 4): 8
Enter the lower limit 'a': 0
Enter the upper limit 'b': 1
The integral is: 0.693148
Process returned 0 (0x0)   execution time : 4.360 s
Press any key to continue.
```

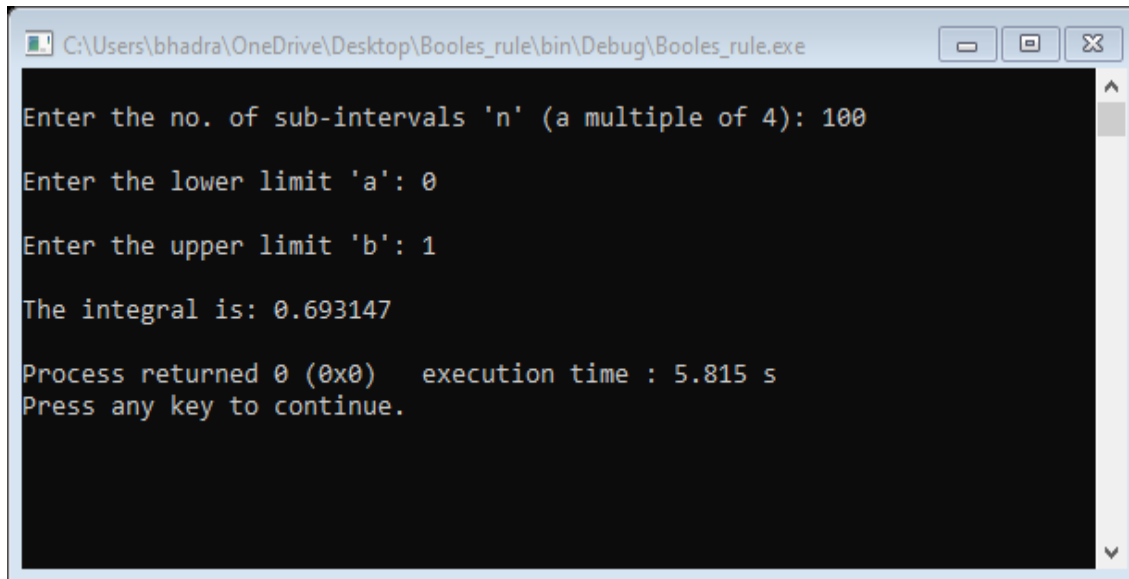
Screenshot showing the integral is not applicable if n
is not a multiple of 4:



```
C:\Users\bhadra\OneDrive\Desktop\Booles_rule\bin\Debug\Booles_rule.exe

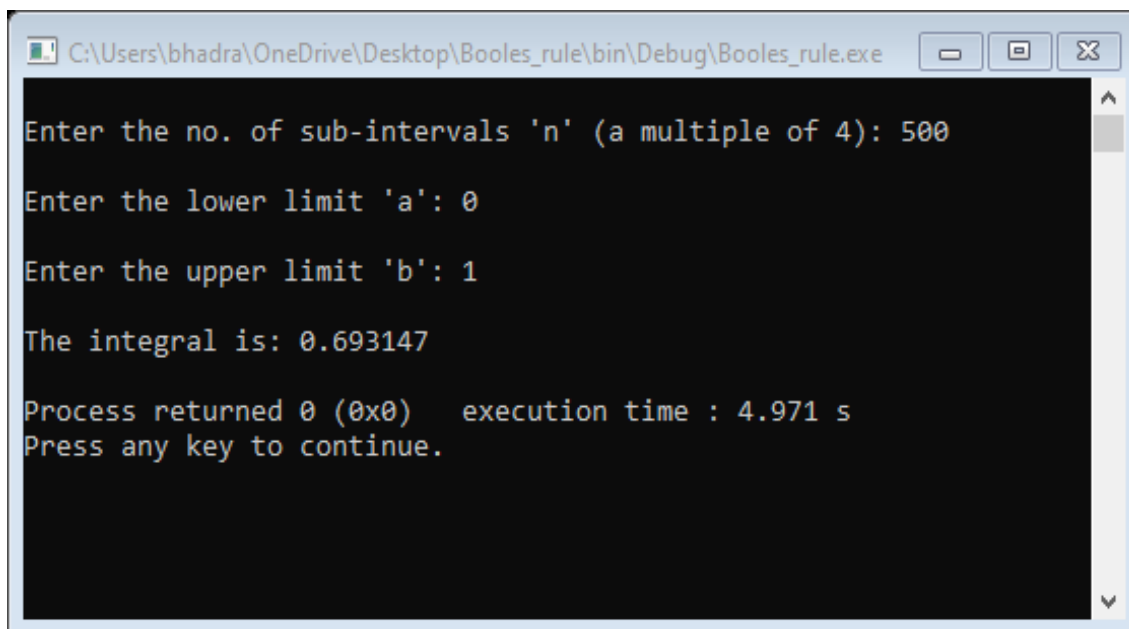
Enter the no. of sub-intervals 'n' (a multiple of 4): 6
Oops!
BOOLE'S RULE is not applicable.
Please enter a number which is a multiple of 4.
Process returned 0 (0x0)   execution time : 2.366 s
Press any key to continue.
```

Screenshots of the output showing integral value 0.693417 which matches with the analytical solution with $n = 100$:



```
C:\Users\bhadra\OneDrive\Desktop\Booles_rule\bin\Debug\Booles_rule.exe

Enter the no. of sub-intervals 'n' (a multiple of 4): 100
Enter the lower limit 'a': 0
Enter the upper limit 'b': 1
The integral is: 0.693147
Process returned 0 (0x0)   execution time : 5.815 s
Press any key to continue.
```



```
C:\Users\bhadra\OneDrive\Desktop\Booles_rule\bin\Debug\Booles_rule.exe

Enter the no. of sub-intervals 'n' (a multiple of 4): 500
Enter the lower limit 'a': 0
Enter the upper limit 'b': 1
The integral is: 0.693147
Process returned 0 (0x0)   execution time : 4.971 s
Press any key to continue.
```

WEDDLE'S RULE

Taking $n = 6$ in the the General Quadrature formula, all differences higher than the sixth order will become zero and thus we get

$$\begin{aligned} \int_{x_0}^{x_n} f(x) dx = & \frac{3h}{10} [(y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6) \\ & + (y_6 + 5y_7 + y_8 + 6y_9 + y_{10} + 5y_{11} + y_{12}) + \cdots \\ & + (y_{n-6} + 5y_{n-5} + y_{n-4} + 6y_{n-3} + y_{n-2} + 5y_{n-1} + y_n)] \end{aligned}$$

This is known as *Weddle's rule*. The number of intervals in this rule must be a multiple of 6.

Example Evaluate $\int_0^1 \frac{1}{1+x} dx$ by using *Boole's rule* with six sub-intervals.

Solution Divide the interval $[0,1]$ into six subintervals.

$$\text{Here } h = \frac{b-a}{6} = \frac{1-0}{6} = \frac{1}{6}$$

The values of x and y are tabulated as below:

x	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1
$y = \frac{1}{1+x}$	1	0.857142	0.75	0.666667	0.6	0.545454	0.5

$$\therefore y_0 = 1, y_1 = 0.857142, y_2 = 0.75, y_3 = 0.6667, y_4 = 0.6, y_5 = 0.545454, y_6 = 0.5$$

By Weddle's rule,

$$\int_0^1 \frac{1}{1+x} dx = \frac{3h}{10} [y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6]$$

$$= \frac{1}{20} [1 + 5(0.857142 + 0.75 + 6(0.666667) + 0.6 + 5(0.545454 + 0.5))]$$

$$= 0.693149$$

WEDDLE'S RULE C-PROGRAM

```
#include<stdio.h>
#include<math.h>

double f(double x)
{
    return 1/(1+x);
}

main()
{
    int n,m,i;
    double a,b,h,x,sum=0,integral;

    printf("\nEnter the no. of sub-intervals 'n' (EVEN): ");
    scanf("%d",&n);

    if (n%6==0)
    {
        printf("\nEnter the lower limit 'a': ");
        scanf("%lf",&a);

        printf("\nEnter the upper limit 'b': ");
        scanf("%lf",&b);

        h=(b-a)/n;
        m=n/6;

        for(i=1;i<=m;i++)
        {
            sum=sum+f(a)+5*f(a+h)+f(a+2*h)+6*f(a+3*h)+f(a+4*h)+5*
f(a+5*h)+ f(a+6*h);
            a=a+6*h;
        }
    }
}
```

```

        integral=(3*h/10)*sum;

        printf("\nThe integral is: %lf\n",integral);

    }

else
    {
        printf("Oops!\n WEDDLE'S RULE is not applicable.\n Please
enter a number which is a multiple of 6.\n");
    }
}

===== END OF THE PROGRAM =====

```

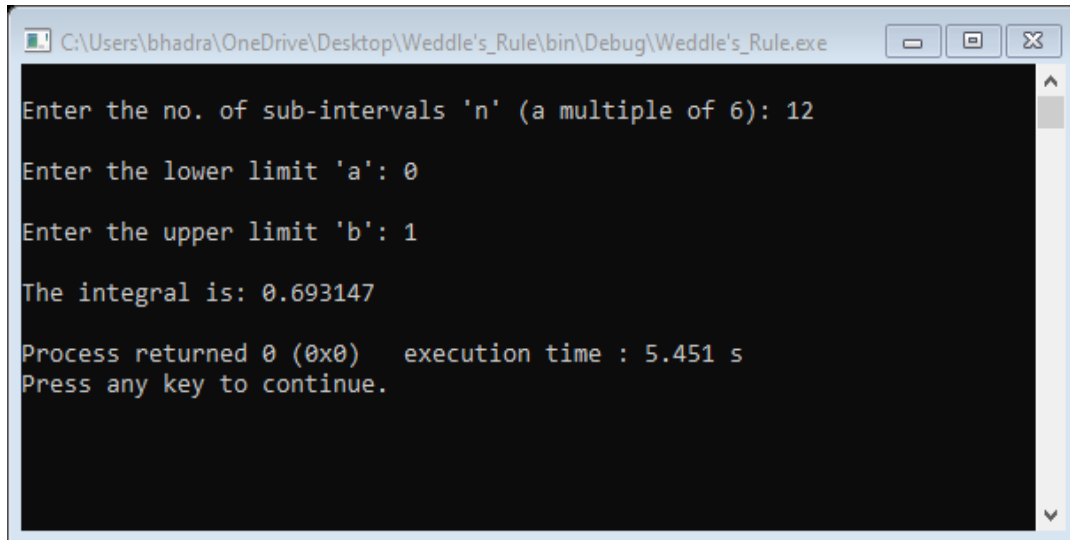
Screenshot of the c-program:

```

1  #include<stdio.h>
2  #include<math.h>
3  double f(double x)
4  {
5      return 1/(1+x);
6  }
7  main()
8  {
9      int n,m,i;
10     double a,b,h,x,sum=0,integral;
11     printf("\nEnter the no. of sub-intervals 'n' (a multiple of 6): ");
12     scanf("%d",&n);
13     if (n%6==0)
14     {
15         printf("\nEnter the lower limit 'a': ");
16         scanf("%lf",&a);
17         printf("\nEnter the upper limit 'b': ");
18         scanf("%lf",&b);
19         h=(b-a)/n;
20         m=n/6;
21         for(i=1;i<=m;i++)
22         {
23             sum=sum+f(a)+5*f(a+h)+f(a+2*h)+6*f(a+3*h)+f(a+4*h)+5*f(a+5*h)+f(a+6*h);
24             a=a+6*h;
25         }
26         integral=(3*h/10)*sum;
27         printf("\nThe integral is: %lf\n",integral);
28     }
29     else
30     {
31         printf("Oops!\n WEDDLE'S RULE is not applicable.\n Please enter a number which is a multiple of 6.\n");

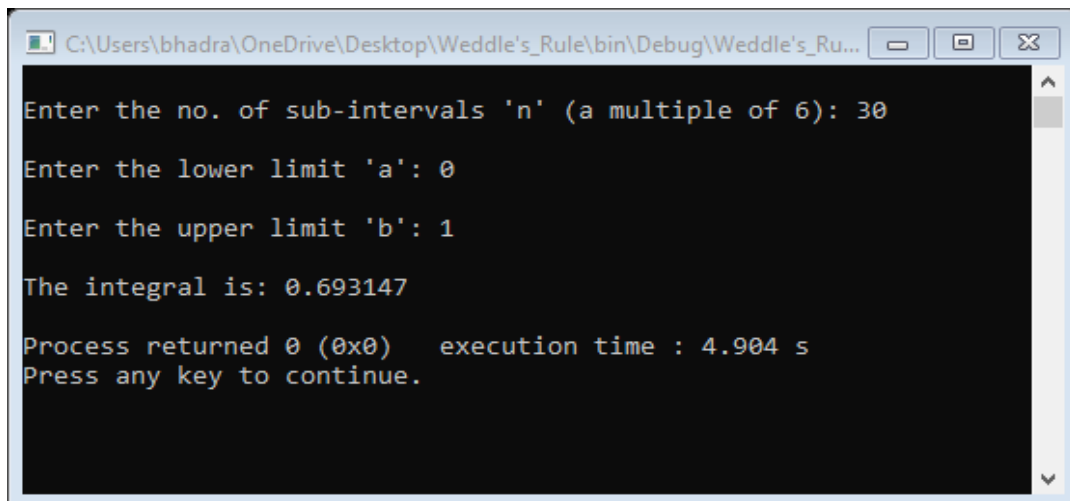
```

Screenshot of the output showing the integral value 0.693147 which matches with the analytical solution with just $n = 6$:



```
C:\Users\bhadra\OneDrive\Desktop\Weddle's_Rule\bin\Debug\Weddle's_Rule.exe

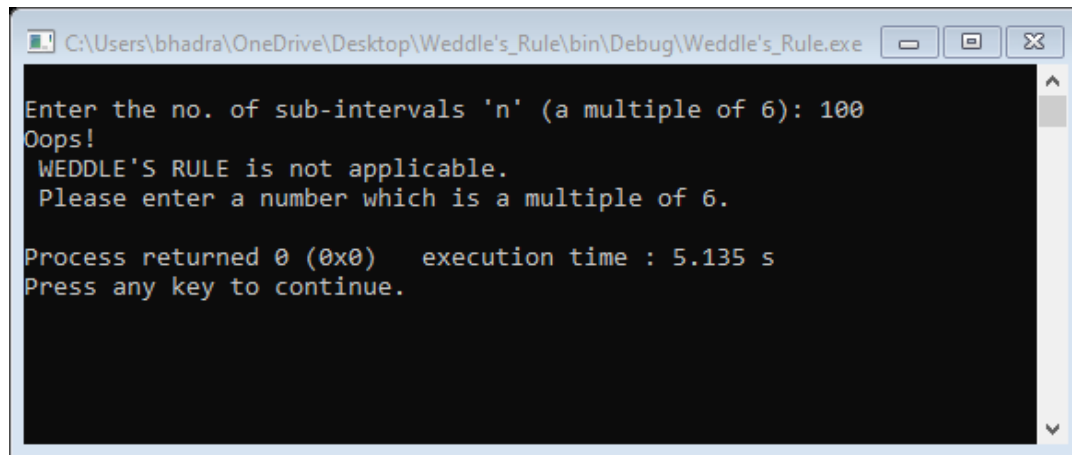
Enter the no. of sub-intervals 'n' (a multiple of 6): 12
Enter the lower limit 'a': 0
Enter the upper limit 'b': 1
The integral is: 0.693147
Process returned 0 (0x0)   execution time : 5.451 s
Press any key to continue.
```



```
C:\Users\bhadra\OneDrive\Desktop\Weddle's_Rule\bin\Debug\Weddle's_Ru...

Enter the no. of sub-intervals 'n' (a multiple of 6): 30
Enter the lower limit 'a': 0
Enter the upper limit 'b': 1
The integral is: 0.693147
Process returned 0 (0x0)   execution time : 4.904 s
Press any key to continue.
```

Screenshot of the output showing that the rule is not applicable if n is not a multiple of 6:



```
C:\Users\bhadra\OneDrive\Desktop\Weddle's_Rule\bin\Debug\Weddle's_Rule.exe
Enter the no. of sub-intervals 'n' (a multiple of 6): 100
Oops!
WEDDLE'S RULE is not applicable.
Please enter a number which is a multiple of 6.

Process returned 0 (0x0)   execution time : 5.135 s
Press any key to continue.
```

CONCLUSION

The value of the integral in Trapezoidal rule matches with the analytical solution with the n value near to 500 and in case of , Simpson's one-third rule and in Boole's rule it is obtained with with the n value near to 100 and with the Weddle's rule it is obtained with just $n = 12$. Thus Simpson's one-third rule is more accurate than the Trapezoidal rule.

Among all the rules, Weddle's Rule is more accurate than any of the other rules.

References

1. C-Language Tutorial videos by Srinivas, Naresh Technologies
https://youtube.com/playlist?list=PLVIQHNRLfIP8IGz6OXwIV_lgHgc72aXlh
2. Finite Differences and Numerical Analysis by H.C Saxena published by S.Chand and Company, Pvt. Ltd., New Delhi.
3. Numerical Analysis by S.Ranganatham, MVSSN Prasad and V.Ramesh Babu, S.Chand Publications.